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Fluid to particle heat transfer in a fluidized bed and to single particles

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Abstract—A frequency response technique to evaluate gas to particle heat transfer coefficients to solid spheres and to fine particle fluidized beds is described. At low Reynolds numbers, the single particle Nusselt number approaches 2 as expected. However, the general magnitude for fine particle fluidized beds is 20–100 times those for packed or single particle systems, in marked contrast to the literature data. The present results are more rational, but still are model dependent as are all the other fluidized bed results.

INTRODUCTION

TURTON *et al.* [1] cited the background and presented a new technique for the measurement of the heat transfer coefficient between fine fluidized particles and air. As they pointed out, there has been extensive work for the last 30 years on the subject. They also pointed out there is little agreement and certainly no rational explanation for the low coefficients as the Reynolds number approaches zero. Shortly after joining Ohio State, this problem became one of interest to us because of its importance in determining the temperature of solids in a fluidized bed and its influence in turn on rates of reaction when that solid is a catalyst. To emphasize how long ago that was, the project was supported by the AEC. Kim [2] completed the work for his Ph.D. in 1965 on the use of a frequency response method to solve the problem. A paper by Kim *et al.* [3] appeared in 1972 on the corresponding packed bed results. In this article, mention was made of using the technique for the fluidized bed. Earlier, Brodkey [4] cited Kim's work on fluidized beds, presented the model used, and mentioned that the coefficients obtained "were considerably greater than the results for single or packed beds at low Reynolds numbers". Because the technique was difficult, it was unlikely that anyone would use the method and publish results; thus, the second phase of Kim's work on fluidized beds lay dormant in his dissertation [2] and that on the single particle heat transfer in the M.S. thesis [5] by Sidner which was completed in 1963. Professor L. S. Fan of our Department brought Professor Levenspiel's and his co-workers work to my attention and I decided to revisit the subject some 25 years later.

As pointed out by Kunii and Levenspiel [6] and Turton *et al.* [1], the heat transfer coefficients or Nusselt numbers from particles to the fluid in a fluidized bed, as measured experimentally, are lower than the theoretical minimum (Nusselt number of 2) for a single spherical particle at low Reynolds numbers. The difference is not trivial in that at the lowest Reynolds numbers, the difference can be three orders of magnitude. In this work, an attempt is made to resolve this inconsistency by studying the fluid–particle heat transfer of single particles and of fluidized bed systems by using a frequency response technique. The main emphasis was placed on the theoretical formulation and the development of the experimental method. However, enough experimental data were obtained for the intended purpose of rationalizing the apparent inconsistency.

Although Turton *et al.* [1] cited the background for this work, it is necessary to briefly review the literature because of the gap in time. Several key references were cited in our earlier work [3]. Lindauer's [7] work on packed and fluidized beds at high Reynolds numbers was reviewed and he cited the Kim effort. Lindauer used shallow beds ($L/D_p < 7$) and found that h was independent of the mass velocity, which is contrary to packed-bed experience. His values agreed with the packed-bed results which is to be expected for the Reynolds number range considered (>100). The troublesome low values are at low Reynolds numbers in the range of 40 and below. The work of Littman and his co-workers dates from the same period as that of Kim. In fact, the Kim facilities were later contributed to Professor Littman's program, when no further work was planned. The efforts by Littman and Stone [8] and Littman and Barile [9] were cited by Kim from the original thesis by Barile. Because of the model used and the fact that the fluidized bed is very close to a stirred tank, only minimum values for h could be obtained. Unfortunately, these were close to the low values cited earlier. Thus, the model cannot resolve the heat transfer problem. The models used by all these researchers were very similar (including the packed-bed model used by Kim *et al.* [3]) and had Lindauer had low Reynolds number data, they would have been lower than reasonable.

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NOMENCLATURE

A	area, amplitude of temperature vector
c_p	heat capacity
C	constant
D_p	particle diameter
e	fraction of solids in the lower zone
h	local heat transfer coefficient
i	$\sqrt{-1}$
k	thermal conductivity
M	mass
N_{Nu}	Nusselt number, hD_p/k_g
N_{Re}	Reynolds number, $D_p u \rho_g/\mu$
q	rate of heat transfer
R	amplitude ratio
S	interchange of solids between upper and transfer zones
t	time
T	temperature
u	actual gas velocity (superficial velocity/void fraction)
w	mass flow rate
x	distance along wire from particle
z	vertical distance from the support.

Greek symbols

Δ	difference
ε	void fraction
θ	time constant
θ_1	$M_s c_{ps}/hA$
θ_2	$M_s c_{ps}/w_g c_{pg}$
μ	viscosity
ρ	density
ϕ	phase shift
ω	frequency.

Subscripts

co	copper
cu	constantan
g	gas
i	inlet
o	outlet
s	solid
t	lower transfer zone
u	upper equilibrium zone.

Gunn [10] and Gunn and Narayanan [11] also treated the frequency response problem. Gunn's [10] first effort was theoretical and did not address the fluidized bed problem. The more recent work by Gunn and Narayanan treated dispersion and heat transfer in fluidized beds. Their results are the highest reported until our work (see ref. [1]) and agreed closely with those for a packed bed. They are higher than the low results cited previously. Gunn attributed this difference to the neglect of the axial thermal dispersion by others. They are, however, much lower than the results to be reported here.

The results of our single particle heat transfer experiments by Sidner [5] verified the theoretical minimum, as others had done, and thus established the validity of the frequency response method. His work allowed data to be obtained down to Reynolds numbers near 2, an order of magnitude below other results at that time. The theoretical analysis by Johnston *et al.* [12] shows that the minimum corresponds to a Nusselt number of 2. A rational thought analysis suggests that the number cannot be less than 2 for the fluidized bed also. The effect of adjacent particles on the heat transfer from a given particle is not well established, but it is agreed that the presence of neighboring particles facilitates the transfer. In a fluidized system, there are many possible effects. They are: reduction of the available area for convective transfer due to contact, removal of the boundary layer due to collision, and generation of turbulence by the motion of bubbles and collisions of the particles. All of these effects, except area reduction, facilitate transfer. Some quantitative information about the relative significance of these effects can be derived from packed-bed experiments. Since packed-bed results exceed the minimum Nusselt number and the packed bed can be pictured as a single large agglomerate, area reduction is not sufficient to reduce the Nusselt number below 2. In the fluidized bed some of the solid moves with the stream at a lower relative velocity, but since there is no net solid transport some of the solid must move opposite to the stream at a higher relative velocity. If one considers an element of solids under either condition as a small packed bed, the local Nusselt number must be greater than 2 at all points and thus everywhere. Because of the number of difficulties associated with the measurements and the interpretation of results, one must conclude that previous authors have not measured the *true* heat transfer coefficient. The word *true* must be emphasized; previous work has measured heat transfer coefficients, but these are limited by the models used. The results prove that such models are inadequate to describe the *actual* mechanism of the transfer of heat in a fluidized bed. However, one should remember that if the heat transfer results for the experiments are used with the model used to evaluate that result, correct estimates of the heat transfer can be made.

Kim *et al.* [3] used a local linear approximation (in both time and space) to analyze successfully the heat transfer coefficients for packed beds. However, this approximation was unsuccessful for the fluidized beds, due to the limited accuracy of the data. As an alternate over-all approach for fluidized beds, various models were constructed on the basis of the physical picture of the bed and the use of simplifying assumptions. The solutions from the resulting differential equations were evaluated with the experimental results. The best agreement and most rational results were obtained with a new two-zone model [2, 4]. The two-zone model is one in which there are two distinct zones (upper and lower) and must not be confused with the two-phase model (emulsion and bubble). Since the results were more rational, one can conclude that the suggested model may be closer to the truth, but there is no guarantee that even this is true.

The local heat transfer between the gas and a particle in a fluidized bed cannot be studied easily at steady state because there is no convenient way to determine the amount of heat transferred or the particle's temperature. Unsteady-state experiments in which the particle's temperature could be measured would allow the determination. This is a point strongly made by Turton *et al.* [1]. An unsteady-state experiment was used in this work also, but rather than use a transient, we used a sinusoidal temperature variation input so the phase shift and amplitude decay could be measured to a high degree of accuracy.

THEORY

A heat transfer process can be expressed by the simple rate equation, $q = hA\Delta T$. All terms except A may be functions of time. The heat transfer coefficient, h, is a function of the other variables.

The use of the solid surface area, A, should be correct for a study of the particle to gas heat transfer. However, is it the full solids area of the bed or just some specific part? This is a major source of disagreement and may well be the main cause for the existing difference between the reported results and those expected. We will show that thermal equilibrium between the particles and surrounding gas is essentially complete at a very short distance above the distributor. However, the total particle surface area in the bed is more often used as the area rather than the active area.

The determination of the heat transferred is relatively simple, if q represents the total amount of heat transferred, and if there are no heat losses. In contrast, for a local differential volume, the heat transferred can be obtained only by indirect methods which necessitate an accurate knowledge of mixing.

The temperature difference is the most troublesome term in the equation. The driving force, ΔT, must be defined consistent with the selected area. For measurement of the gas temperature, a suction tube and a high-speed thermocouple, are adequate. However, there are dangers which must be avoided. When the suction is too high, it disturbs the flow pattern of the bed, and any solids which catch on the shield can change the true gas temperature. These effects are hard to evaluate. Another danger is that there is no way of telling whether the gas is from the bubble or emulsion. If the suction probe is large, gases of both phases enter simultaneously. The most controversial term is the solid temperature. Leva [13] indicated that accurate determination of this solid temperature would solve the problem of fluid-bed heat transfer. Wamsley and Johanson [14] adopted an unsteady-state experiment to eliminate the necessity of using the solid temperature. It is this concept that holds the most promise [1, 2, 5] and is used here.

Two-zone model

The *two-phase model* that uses a gas as bubbles and an emulsion phase has found widespread use in the analysis for fluidized beds, especially when kinetics are involved. However, that model has not been of value for the particle to solid heat transfer problem. It is important to note that the heat of the gas per unit volume is several orders of magnitude less than that of the solid. Thus it takes little solid to cool a large gas flow. The rapid cooling of a gas stream has been well documented by Heertjes *et al.* [15, 16], who showed that cooling was essentially complete in the first 1/4 in. above the distributor. Kim [2] confirmed these results. It is this same region that is low in solids because of the jetting action of the grid [17] and is below the region where bubbles form. Above this region, bubbles form and mixing of solids is intense; thus, the assumption of complete mixing should be adequate. The *two-zone model* is illustrated in Fig. 1. This model is based on the hypothesis that cool solids come into contact with the entering hot gas in the transfer zone and are heated. By the solids mixing process, the now heated solids are returned to the upper zone and are rapidly mixed. Further transfer from the solids to the gas occurs here; however, since the solids have not been heated drastically (the heat content difference), and since those that are heated are mixed into a much larger body of solids, we would not expect to see a really measurable temperature difference here. This does not mean that equilibrium exists, as has been assumed in many previous models for heat transfer. It means, rather, that finite heat transfer occurs and cannot be ignored, because even

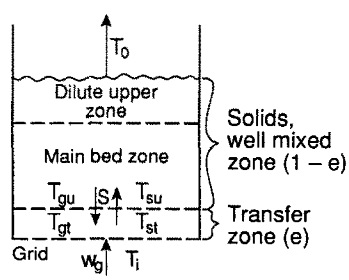

FIG. 1. Two-zone model for heat transfer in a fluidized bed: T_s and T_g are the solid and gas temperatures; subscripts –t and –u refer to the transfer zone and the upper zone; T_i and T_o the inlet and outlet temperatures; e the fraction of solid in the transfer zone; S the interchange of solids between the upper and lower zones; and w_g the mass flow rate of gas.

though the temperature driving force is very small, the area for transfer is very large. An overall heat balance on the system results in

$$w_g c_{pg}(T_i - T_o) = M_{gt} c_{pg}(dT_{gt}/dt) + eM_s c_{ps}(dT_{st}/dt) + M_{gu} c_{pg}(dT_{gu}/dt) + (1-e)M_s c_{ps}(dT_{su}/dt) \quad (1)$$

where the notation from ref. [4] is used (shown in Fig. 1). For the solids

$$eh_t A(T_{gt} - T_{st}) + (1-e)h_u A(T_{gu} - T_{su}) = eM_s c_{ps}(dT_{st}/dt) + (1-e)M_s c_{ps}(dT_{su}/dt). \quad (2)$$

To simplify the problem, we can assume:

(1) The thermal capacitance of the gas is negligible in both zones or $M_{gt}(dT_{gt}/dt) = M_{gu}(dT_{gu}/dt) = 0$.

(2) Mixing of solids is great so that $T_{st} = T_{su} = T_s$.

(3) The gas temperature in the upper zone is equal to the outlet gas temperature $T_{gu} = T_o$.

(4) The gas temperature in the lower zone is equal to the inlet gas temperature $T_{gt} = T_i$.

The last assumption is the weakest, since it is known that there is a gradient in this area. Equations (1) and (2), with the additional assumption that e is very close to unity, become

$$w_g c_{pg}(T_i - T_o) = M_s c_{ps}(dT_s/dt) \quad (3)$$

and

$$M_s c_{ps}(dT_s/dt) = eAh_t(T_i - T_s) + (1-e)Ah_u(T_o - T_s) \quad (4)$$

where two separate values of h are used to signify the marked physical difference of the two zones and the + sign on the last terms for the case of gain of heat by the solid in the upper zone.

Frequency response method

Successful and extensive application of the frequency response method for characterization of the time dependence of complex electronic networks is well known. However, its application to a characterization of flow systems, where the additional effect of location is involved, was relatively new at the time of this work. For fluidized beds, sinusoidal variation of the inlet temperature has been used by refs. [2, 7]. A square wave input was used by refs. [8, 9] and a pulse by ref. [11]. Sinusoidal variation provides better accuracy because harmonics need not be considered; however, this is at the expense of a more complex inlet temperature generator. Recent applications in the fields of communication and control, and especially for the characterization of continuously stirred reactors, can be found in any text on control.

For the sinusoidal variation method, a signal with certain amplitude and frequency is impressed on the system under investigation and a resulting output signal from the system with a different amplitude and phase but with the same frequency (except for the special case where the system has its own natural frequency) is obtained. The signal can be any physical quantity which corresponds to a driving force: such as voltage, temperature, or concentration. The relationship of two amplitudes and phase changes at the various frequencies are used to evaluate the differential equation (or transfer function) which represents the system. When the differential equation of the system is linear, the ratio of amplitudes and their relative phase are functions of the frequency only. The intended purpose can be either a selection of the proper differential equation, a determination of coefficients of an equation with known form, or both.

This method is particularly suited for this study, where the objective was not only a selection of the differential equations which most closely describe the fluidized and packed bed systems, but also for the determination of heat transfer coefficients or Nusselt numbers, which represent each mechanism of transfer.

As will be seen, phase angles very close to 90° had to be measured, which was difficult. The procedure used is given in Appendix A. Nevertheless, measurements could be made, and the Nusselt numbers obtained approached 2 for the single particles and were considerably greater than the results for single or packed beds for the fluidized bed at low Reynolds numbers. Since a wide range of variables was not investigated so no new correlation was attempted.

EXPERIMENTAL APPROACH

Among the various modes of heating or cooling used to cause heat transfer to or from particles to a gas, the use of a sinusoidal inlet gas temperature provides a system with a minimum of disturbances. Possible shortcomings of this method are the disturbances due to natural convection. The system components will be briefly described here; more detail can be found in ref. [2].

Sinusoidal power generator

The sinusoidal power generator is shown in Fig. 2. Two powerstats (type 136) were connected in series; one was used to set the amplitude of input to the other

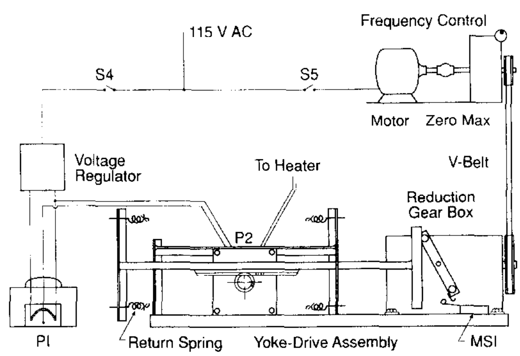

FIG. 2. Sinusoidal power generator; S, switch; MS, microswitch; P, powerstat.

powerstat, which was driven by a scotch yoke. The motion of the yoke originated from a 1/4 hp motor connected to a torque converter, which in turn provided a continuous variation in the output r.p.m. The output was belt-connected to a speed-reducer, the output of which drove the scotch yoke with two identical half-cycles (with spring actuated return). This system generated sinusoidal power with a frequency range of 0–1.5 cycles per minute and an r.m.s. voltage range of 0–100. The stability of frequency was excellent; no change in frequency was observed during 8 h of continued operation. The deviations in the voltage amplitude from pure sinusoidal voltage, if they existed, were very slight.

Test bed

The test bed (shown in Fig. 3) can be divided into three sections: calming zone, heating zone, and bed. The calming zone was made from 12 cm (4.75 in.) i.d. and 17.8 cm (7 in.) long steel tubing. The air inlet was located at the center of a closed bottom section. To provide a uniform inlet gas velocity profile, two porous stainless steel plates and five layers of closely knit nylon cloth were either cemented to the wall or stretched by tension rings. The uniformity of the velocity was checked with a hot film anemometer and found to be excellent. The heating zone consisted of two heater plates, insulators, and gaskets. The heating element was a very thin nichrome ribbon. Two plates were connected in series and had a total resistance of 55 Ω. The time constant of the heater at the maximum gas flow rate was about 3.0 s. The bed was a vacuum jacketed pyrex tube with a cross-section of 111 cm^2 (0.120 $ft.^2$). Three materials—cotton, nylon, and copper—were investigated for the bed support. A thermally and electrically insulated photoetched 400 mesh copper microscreen support was selected. A thin (40 gauge) constantan wire was connected to the screen to form a junction for the screen temperature.

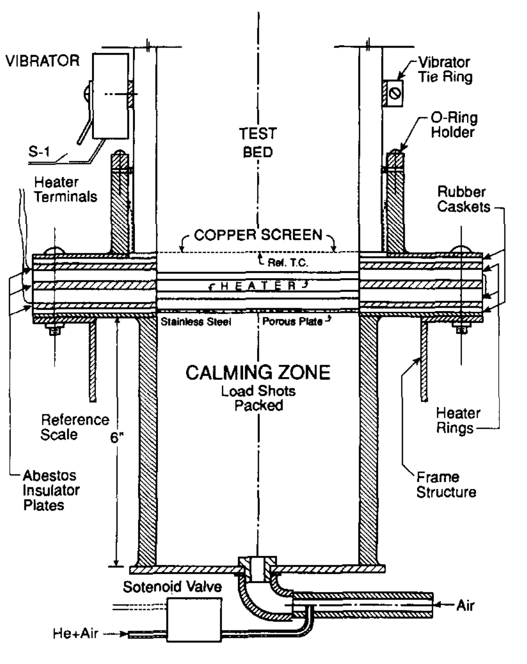

FIG. 3. Bed-heater-calming section detail of system: S, switch.

For the gas inlet to the test bed, compressed air was passed through a fiber-glass packed filter which also served as a surge tank. Two needle valves were used to give a constant pressure to the rotameter which was calibrated for air under 2 atm and 27.8°C (82°F). Air leaving the second valve went to the calming zone. The maximum capacity of the rotameter was equivalent to a superficial gas velocity of 0.43 m s^{-1} (1.42 ft. s^{-1}) in the bed. For an accurate measurement of low flow rates in the bed, a small replacement float was used.

Gas temperature measuring system

The gas temperature was measured with a suction tube thermocouple. The vacuum was maintained in a 19 liter (5 gallon) jar by the use of a vacuum pump. The gas flow rate through the probe was regulated at the rotameter. The voltages from the thermocouples were connected to a terminal board. Each pair of terminals was wired to two identical selector switches. The output of the switches was put through two attenuators (ten turn potentiometers) and then to a patch board. The board was used to add or subtract two signals from the attenuators for the accurate measurement of phase shift between two original signals. The signals from the board were recorded on an Offner two channel Dynograph recorder which has a sensitivity of 1 μV per millimeter of chart. A 0.476 cm (3/16 in.) stainless steel plate served as a cover for the bed and as the probe holder, and also provided for the horizontal coordinate by the sliding of the plate. The probe holder could be moved vertically within a range of 0.35 m (14 in.). The accuracy of positioning was better than 0.5 mm.

The single particle heat transfer experiments

The frequency response method was also used for the determination of the heat transfer from a single particle. A 40 gauge copper–constantan thermocouple was imbedded in a 1.0 mm solder ball for the solid particle temperature. A bare thermocouple was used to indicate the surrounding gas temperature. The same test bed, but empty, was used to generate the sinusoidal temperature field.

EXPERIMENTAL RESULTS

Empty bed experiments

A series of runs were made in the empty bed to check the uniformity of the heat flux over the cross section, the uniformity of vertical gas velocity, the effect of natural convection, and the heat loss. The results indicated that the uniformity in heat flux and the gas velocity at the lower section of the bed was very satisfactory. At a distance of 0.254 m (10 in.) above the support, deformation of the sine wave was so great that the frequency and amplitude could not be reproduced. The height at which the wave starts to deform varied with the gas flow rate, input frequency, and amplitude of the input. At the lower gas flows, higher frequency and larger amplitudes decreased the height. From these observations it was concluded there were strong effects of natural convection, and consequently mixing in an empty tube. This does not mean that strong effects of natural convection will exist in a packed or fluidized bed, since viscous effects due to high surface area would be much greater than buoyancy, and the actual vertical gas velocity would be much higher due to the decrease in voids.

The average gas temperature in the empty bed was constant, regardless of height. This indicates there were no net heat losses through the wall. Some dampening of the amplitude should exist in the area near the wall because of the possible transfer between wall and gas; this was detected.

Single particle heat transfer experiments

The single particle heat transfer experiments were intended (1) to extend the existing N_{Nu} vs N_{Re} correlation of single particle heat transfer to lower Reynolds numbers, (2) to characterize the solder ball thermocouples which were used in the fluidized bed heat transfer experiments, and (3) to test the theory and the technique of using the frequency response approach.

These experiments were carried out in the empty bed. The solder ball thermocouples used in this study were 40 gauge copper–constantan thermocouples embedded in solder balls whose diameters were about 1 mm. Evaluations of the heat transfer coefficients were made from the measured time constants, ball diameters, and physical properties of the solder. A correction for the effect of the lead wires was also applied. The magnitudes of this correction and the convective disturbance of the wave at low gas flow rates limited the experiments to a lower Reynolds number of about 2. The results clearly showed that heat transfer of very low flows approaches a Nusselt number of 2.

Theory for treatment of data. The use of a sinusoidal inlet gas temperature allows analysis by a simple frequency response technique. The gas and solid temperatures are a function of ω and t

$$T_g = A_1\,e^{i\omega t} \quad \text{and} \quad T_s = A_2\,e^{i\omega t+\phi}. \tag{5}$$

A heat balance on a single particle gives

$$M_s c_{ps}(\partial T_s/\partial t) = hA(T_g - T_s)$$

$$\text{or} \quad \partial T_s/\partial t = (1/\theta_1)(T_g - T_s) \tag{6}$$

where θ_1 = time constant of this particular system $= M_s c_{ps}/hA$. Substituting equations (5) for T_g and T_s into equations (6) gives

$$[\partial(A_2\,e^{i\omega t+\phi})/\partial t] = (1/\theta_1)(A_1\,e^{i\omega t} - A_2\,e^{i\omega t+\phi})$$

or

$$i\omega\theta A_2\,e^{i\omega t+\phi} + \phi = A_1\,e^{i\omega t} - A_2\,e^{i\omega t+\phi} \tag{7}$$

which gives

$$T_g - T_s = i\omega\theta_1 T_s. \tag{8}$$

Thus the gas temperature can be represented as a vector with the solid temperature lagging at some phase angle ϕ. The vector difference between the two temperatures is $i\omega\theta_1 T_s$. Since at steady state (no sinusoidal input) $T_g = T_s$, the two thermocouples were adjusted to give the same output values under steady-state conditions by potentiometers. The solid temperature T_s, and the temperature difference were measured under experimental conditions.

Since $|T_g| - |T_s|/|T_s| = \omega\theta_1$, it was only necessary to divide by ω to determine the time constant. Simple substitution gives the Nusselt number

$$\theta_1 = M_s c_{ps}/hA \quad \text{or} \quad h = M_s c_{ps}/\theta_1 A \tag{9}$$

$$N_{Nu} = hD_p/k_g = M_s c_{ps} D_p/\theta_1 \pi D_p^2 = M_s c_{ps}/\theta_1 \pi D_p. \tag{10}$$

The particle was weighed by a micro-balance and the diameter was determined. Heat capacity data were obtained from handbooks. The Reynolds number was determined from the fluid properties, the particle diameter, and the velocity. An analysis was necessary to enable subtraction of the contribution of the copper and the constantan leads from the effective heat transfer coefficient. The analysis can be found in Appendix B.

Results and discussion of single particle heat transfer experiments. The results for runs on three spheres (0.935–1.14 mm) can be found in ref. [5]. For a given flow rate the time constant should remain constant regardless of the frequency. The data from ref. [5] show that this is true, thus average values could be used. The resulting Nusselt numbers calculated from these time constants are compared to the predicted values in Fig. 4. Good agreement is noted except at the lowest flow rates. Several sources of error can lead to these differences. A dampening and distortion of the measured sine curves would indicate the presence of convective effects. Density differences would cause free convection to contribute to the heat transfer rate. At higher velocities this effect can be neglected. At the lower rates, gravitational forces are not counteracted, and thus can lead to convective effects.

A primary source of error might be in the correction

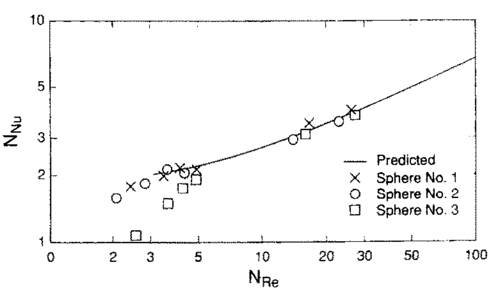

FIG. 4. Nusselt vs Reynolds numbers for single particles.

for the contribution of the leads. This correction is large, equaling as much as 75% of the total heat transferred. The correction is greater at the lowest flow rates and certainly can lead to the low values of observed Nusselt numbers, since a 10% error in the lead correction may result in a 30% error in the contribution of the particle. In determining the lead correction the surface temperature of the solid was taken as T_s to avoid a split boundary analysis. Any difference in the slope at the interface caused by an internal temperature gradient will also lead to error.

Data obtained on the spherical particles shows that in single-particle heat transfer, the Nusselt number will approach 2.0 at low Reynolds numbers as predicted by theory. Thus, an imbedded thermocouple is useful for solid temperature measurement in a sinusoidal system using frequency response techniques.

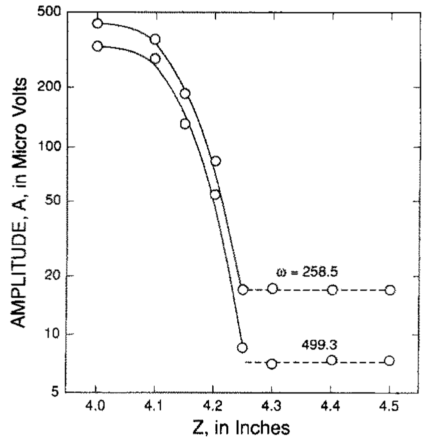

FIG. 5. Amplitude vs height for one typical fluidized bed experiment.

Fluidized bed heat transfer experiments

The preliminary runs and general comments. A series of fluidized bed heat transfer experiments were conducted to study the general qualitative nature of the system and to check the various temperature measuring devices. These were used to test the linear flow models and to develop theoretical models for the description of the system. In addition, two sets of mass transfer experiments were undertaken to determine the general degree of gas backmixing. These will not be reported here, but can be found in ref. [2].

Based upon the measured gas temperature waves, a fluidized bed system can be classified into three general modes by the level of the gas flow rate. When the fluidizing gas velocity is low so only infrequent and discrete bubbles are observed in the bed, the plots of log amplitude decay and phase shift against the distance from the distributor showed a strong similarity to those obtained in the packed bed experiments [3]. There was a small scatter of individual waves about some mean values as the bubbles pass by the tip of the probe. As the gas flow rate increased, the size and frequency of the bubbles tended to increase. A slow gross motion of solids was also observed. The measured gas waves were sufficiently distorted in their frequency and amplitude so that the waves could not be characterized. This distortion was caused by the interaction of the impressed sine wave and the oscillatory nature of the system, caused by the gross motion of solid. This distortion was everywhere in the bed, but its extent was different. In this range of gas flow rates, a further increase of the gas flow lessens the amplitude disturbance and increases the frequency. As a result, a gradual improvement of the temperature waves was obtained. However, this system still lacked the general reproducibility which was essential for a detailed experiment. Finally, when the gas flow rate was increased to the extent that a violent motion of solids and bubbles was observed, the lines of the two plots were quite different (see Figs. 5 and 6). These results indicated that the dampening of amplitudes and the phase shift occur within 0.64 cm (0.25 in.) from the screen and that a near-or-at-equilibrium solid and gas temperature exists in the rest of the bed. This must be caused by the violent circulation and mixing of solid particles in the equilibrium section of the bed.

In fluidized beds with bubbles, regardless of the gas flow rate, no general linearity similar to a packed bed was found. In a well fluidized system, however, an apparent linear response of time exists between the inlet and outlet gas temperatures as a result of high frequency and low amplitude of the disturbance. This does not imply that every part of the system obeys the linear criteria of the input–output relationship. Thus, the gas temperature measurement inside the bed is essential and any analysis of the input–output response would be an approximation. This obviously could be misleading.

Two-zone model. The foregoing results of gas temperature measurements in the fluidized bed showed

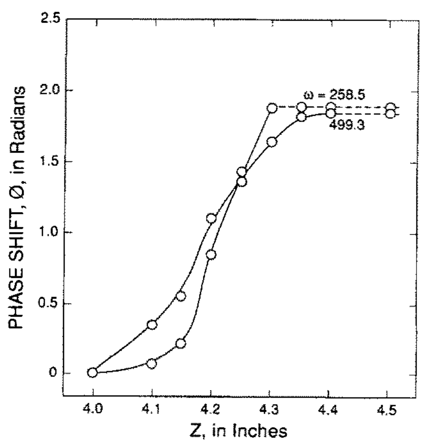

FIG. 6. Phase shift vs height for one typical fluidized bed experiment.

that it could be divided into two characteristic zones; i.e. a decaying zone (0.64 cm or the first 1/4 in. of the bed) and a constant gas temperature zone (the rest of the bed). In the lower portion of the decaying zone, the waves were very sinusoidal but more distorted as the distance from the support was increased. The distortion and scatter were most pronounced at the area near the interface of the two zones. In the decaying zone, the recorded amplitude of the gas temperature was strongly affected by the probe suction rate, the higher suction giving higher amplitudes.

In the constant gas temperature zone, the recorded sine waves had a slight superimposed random scatter but they were very stable and reproducible. Changes in suction rate did not affect the waves in this zone. Identical waves were recorded everywhere in this zone and were equal to the exit gas temperature wave. This was taken as an indication of uniform gas and solid temperatures in the zone. If the mixing rate of gas was extremely high, a minute but uniform driving force (temperature difference between the gas and the solid) could exist. To substantiate the results of the suction probe, use was made of mass transfer experiments, bare thermocouples, a solder thermocouple probe, and a copper screen thermocouple. The results of the mass transfer experiments [2] indicated that waves were very sinusoidal near the bottom of the bed but became very irregular and irreproducible as the distance from the support increased. The height at which a noticeable deformation started corresponded to that in the temperature variation experiment. A bare 40-gauge thermocouple inserted into the bed gave similar results to those of the suction thermocouples, i.e. a rapid decay of amplitude near the screen and an equal temperature in the rest of the bed. Two solder ball thermocouples having widely varied time constants were inserted in the upper section of the bed. Both thermocouples indicated identical temperatures, thus indicating an extremely high rate of interfacial transfer. These thermocouples did show a slightly different temperature in the decaying zone. The recorded temperature of the supporting copper screen was little different from that of the reference thermocouples. This and the results of solder ball experiments confirm the findings of Bakker and Heertjes [18], i.e. the presence of high porosity near the screen. The particles near the screen may be less mobile and less dense because of the jetting air from the openings of the screen and the limitation imposed by the screen. Our qualitative experiment with the use of an optic fiber light probe also showed that the optic density near the screen was considerably less than that in the other portions of the bed.

Based upon these various observations, the fluidized bed should be divided into two zones. A small but finite temperature difference between the gas and the solids may exist in the upper (near equilibrium) zone. It should be emphasized that since this zone contains most of the area of the bed, even a small temperature difference here becomes most significant.

Results and discussions of fluidized bed heat transfer experiments

Glass micro-spheres were selected to obtain the quantitative data which could be used to establish the mechanisms of transfer and to obtain the values of the heat transfer coefficient. These particles were the largest available which can be effectively fluidized in our equipment. The glass spheres had a density of 2.83 and a specific heat of 0.186. The beads used had an average diameter of 1.19×10^{-3} ft. The batch was a mixture of 11.6% 900 μm diameter–48.5% 365 μm diameter–35.0% 325 μm diameter–1.5% 275 μm diameter.

The results indicated that the angle between the vector $(T_i - T_o)$ and T_o was always about 90 deg and that values of R/ω, the amplitude ratio between two vectors per frequency, were almost constant for a given run.

Equation (3) can be rewritten as

$$(T_i - T_o) = i\omega\theta_2 T_s \tag{11}$$

where

$$\theta_2 = M_s c_{ps}/w_g c_{pg}. \tag{12}$$

This equation indicates that the solid temperature also should lag $T_i - T_o$ by 90 deg and the amplitude ratio be constant, θ_2.

If the angle was exactly 90 deg, the solids serve purely as a heat sink, and equation (4) is reduced to

$$M_s c_{ps}(\mathrm{d}T_s/\mathrm{d}t) = eAh_t(T_i - T_s) \tag{13}$$

or, by combining this with equation (3) gives

$$w_g c_{pg} = eAh_t. \tag{14}$$

Unfortunately, this implies that the heat transfer is only dependent on $w_g c_{pg}$. Further, to satisfy the equality, the value of the right-hand side must adjust accordingly, where eA was the actual area of heat transfer. The heat transfer coefficient h_t is dependent on w_g, the gas flow rate, but the form of the dependency as suggested by equation (14) does not seem probable. In this case, no evaluation for h_t was possible.

If the angle is larger than 90 deg, T_s is leading T_o and consequently heat must flow from the solid to the gas in the upper zone while the reverse occurs in the lower zone. The mechanism of transfer can thus be described as fast moving particles picking up heat from the incoming gas at the lower zone and losing the heat to the outgoing gas. This implies that the temperature of the gas going out of the lower zone must be very close to that of the solids, and that the solids mixing is extremely fast. Since practically all of the solids were present in the upper zone, the transfer in the upper zone alone can be expressed from equation (4) as

$$M_s c_{ps}(\mathrm{d}T_s/\mathrm{d}t) = Ah_u(T_o - T_s) = w_g c_{pg}(T_i - T_o). \tag{15}$$

Table 1. Results of fluidized bed heat transfer experiments

Run number	θ_1 Angle	θ_1 Slope	Nusselt number Angle	Nusselt number Slope	Reynolds number
1	0.12×10^{-5}	5×10^{-5}	407	91	8.85
2	7.65×10^{-5}	3×10^{-5}	60	152	8.85
3	7.3×10^{-5}	3×10^{-5}	62.5	152	8.85
4	—	2×10^{-5}	—	228	6.65
5	1.35×10^{-5}	4×10^{-5}	340	114	6.65
6	2.24×10^{-5}	5×10^{-5}	204	91	6.65

The solutions of this equation are

$$R = (\theta_1/\theta_2)\omega(1+\omega^2\theta_1^2)^{-1/2} \text{ and } \tan\phi = -(1/\omega\theta_2) \quad (16)$$

where (as defined in equation (6))

$$\theta_1 = M_s c_{ps}/hA = (c_{ps}\rho_s d_p^2/6k_s N_{Nu}). \quad (17)$$

Since this final model describes the results adequately, all results are analyzed by the above solutions. The first solution was rewritten as

$$(R/\omega)^2 = (\theta_1/\theta_2)^2 - \theta_1^2 R^2 \quad (18)$$

and the values of θ_1^2 were obtained from the slope of R^2/ω^2 vs R^2. These and the results from the angle are shown in Table 1. An error analysis for the angle measurement can be found in ref. [2].

The range of values incorporated above are shown in Fig. 7. The value of ε was calculated and ranged from 0.3 to 0.5. This was used to establish N_{Re}. When the Nusselt numbers for runs 1–3 were approximated from the literature, they were $N_{Nu} = 5$ for a packed bed with 38% void down to $N_{Nu} = 0.30$ for the fluidized bed. These values are equivalent to $\theta_1 = 5.5\times10^{-4}$ and 9.13×10^{-3}, respectively. These were also equivalent to $\tan\phi = 4.55$ and 0.278 at $\omega = 400$ rad h^{-1}. These values were too low, being of the order of one one-hundredth even if the accuracy of the angle measurement was only about 20′ of arc. Thus, it was safe to conclude that the *real* heat transfer coefficient of fluidized systems are much larger than those of the packed bed based on the normal correlations.

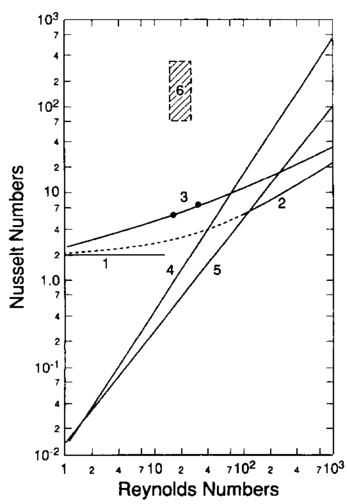

FIG. 7. Nusselt vs Reynolds numbers correlation for fluid to particle transfer: (1) theoretical minimum; (2) single particle; (3) packed beds (ε = 38.3%); (4) fluidized bed, true (Franz); (5) fluidized bed, k apparent (Franz); (6) this work for fluidized beds.

CONCLUSIONS

This work offers an alternate view of the fluidized bed during heat transfer. The two-zone model is the only one to date that can provide particle to fluid heat transfer coefficients that are consistent with common sense that tells us that the values should exceed those for a packed bed.

The results of our study showed that the general magnitudes of the *real* coefficients of fluid–particle heat transfer in fluidized beds at low Reynolds numbers were of the order of 20–100 times those for packed or single particle systems at the same N_{Re}. This result was in marked contrast to data found in the literature [1, 6]. A highly accurate determination was beyond the capability of the present equipment.

The key to the success of the model is that the lower zone is very small when compared to the upper. Its size does not matter as long as equation (4) can be reduced to equation (13). This occurs if e is small when compared to 1 and that the first term is small (because of the factor e) when compared to the second term. With this assumption (and it is an assumption that is the basis of the model), the heat transfer coefficient is determined from using only basic measurements that are in equation (17). These are the time constant (obtained as described in equation (18)), the mass of solids in the bed, the area, and the heat capacity. For the Nusselt number we need to add the solids conductivity, density, and average particle diameter (again note equation (17)). The exchange rate S does not enter and e enters only in the sense that it is assumed small to reduce the model equation as described. The assumptions of the model should be good as long as e is indeed small.

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APPENDIX A. ACCURATE PHASE MEASUREMENT

In the packed bed heat transfer experiments [3], the phase lag ϕ was evaluated by the measurement of the distance between the corresponding peaks (or valleys) of two recorded temperature traces. This method was found to be sufficiently accurate and further improvement in accuracy was attained by an increased number of measurements at varied heights. In contrast, for the fluidized bed heat transfer experiments, the distance measurement could not provide the required accuracy for the data evaluation because the accuracy of the vertical distance measurement of 0.2 mm corresponded to about 10 deg of angle at a high frequency.

As an alternate method, the larger (T_1) of two temperature waves (T_1 and T_2) was attenuated and the resulting wave (T_3) was subtracted from the other (T_2) to obtain a new difference wave (T_4). The phase difference between T_1 and T_2, ϕ, could then be expressed by the following cosine law of the amplitudes:

$$\phi = \arccos\frac{(A_3^2+A_2^2)-A_4^2}{2A_2A_3} \tag{A1}$$

where A is the amplitude for the corresponding temperature vector. Since ϕ was insensitive to the attenuation, many sets of A_3 and A_4 could be obtained, thus allowing improvement of the accuracy which is only limited by the accuracy of the recorder and by the stability of waves.

The magnitude of probable errors in the angle measurement by this method can be expressed by taking the partial derivatives of $\cos\phi$ with respect to each amplitude to be measured. These are

$$\frac{\partial\cos\phi}{\partial A_2} = \frac{1}{A_3} - \frac{1}{A_2}\cos\phi \tag{A2}$$

$$\frac{\partial\cos\phi}{\partial A_3} = \frac{1}{A_2} - \frac{1}{A_3}\cos\phi \tag{A3}$$

and

$$\frac{\partial\cos\phi}{\partial A_4} = -\frac{A_4}{A_2A_3}. \tag{A4}$$

Further, the second terms which contain $\cos\phi$ can be neglected when the angle is near 90 deg. These equations show that the error in $\cos\phi$ which could be caused by each measured amplitude is less than the fraction of error in each amplitude measurement and the total error in $\cos\phi$ is even less than the error by a single amplitude because the right-hand term in the third equation is negative and larger than either $1/A_3$ or $1/A_2$ and less than the sum of the two. The total error becomes

$$\cos\phi \doteq \left(\frac{1}{A_2}+\frac{1}{A_3}-\frac{A_4}{A_2A_3}\right) \approx 1/A \tag{A5}$$

if

$$A_3 = A_2 = A_4.$$

APPENDIX B. CORRECTION FOR THERMOCOUPLE LEADS

An analysis was necessary to enable subtraction of the contribution of the copper and the constantan leads from the effective heat transfer coefficient. The distance x is away from the junction and positive in the copper direction. A heat balance on the wire gives

$$c_pAM_s(\partial T/\partial t) = kA(\partial^2T/\partial x^2)+h\pi D(T_g-T). \tag{B1}$$

If the hold-up is small, the left-hand side of the equation approaches zero, and

$$h\pi D(T-T_g) = kA(\partial^2T/\partial x^2) \tag{B2}$$

$$(\partial^2T/\partial x^2) = (4h/Dk)(T-T_g) \tag{B3}$$

$$[\partial^2(T-T_g)/\partial x^2] = (4h/Dk)(T-T_g) \tag{B4}$$

which may be solved to give

$$T-T_g = C_1\,e^{-(4h/Dk)^{1/2}x}+C_2\,e^{(4h/Dk)^{1/2}x}. \tag{B5}$$

Using the boundary conditions

$$T = T_g \quad \text{as } x\to\infty \quad \text{or} \quad T-T_g = 0,\ C_2 = 0$$

gives

$$T-T_g = C_1\,e^{-(4h/Dk)^{1/2}x} \tag{B6}$$

when

$$x = 0,\ T = T_s \quad \text{or} \quad C_1 = T_s-T_g$$

thus

$$T-T_g = (T_s-T_g)\,e^{-(4h/Dk)^{1/2}x}. \tag{B7}$$

Combining

$$q = kA(\partial T/\partial x)_x = 0 \tag{B8}$$

and

$$(\partial T/\partial x)_{x=0} = -(4h/Dk)^{1/2}(T_g-T_s) \tag{B9}$$

gives

$$q = kA(4h/Dk)^{1/2}(T_g-T_s). \tag{B10}$$

Substituting

$$4h/Dk = h\pi D/kA$$

gives

$$q = (kA\pi Dh)^{1/2}(T_g-T_s). \tag{B11}$$

The total heat transferred equals the actual heat transferred plus the amount transferred by the leads

$$\begin{aligned} q_{tot} &= [hA_s+(k_{cu}A_{cu}\pi D_{cu}h_{cu})^{1/2} \\ &\quad +(k_{co}A_{co}\pi D_{co}h_{co})^{1/2}](T_g-T_s) \\ &= [hA_s+C_1(h_{cu}^{1/2})+C_2(h_{co}^{1/2})](T_g-T_s) \end{aligned} \tag{B12}$$

where

$$C_1 = (k_{cu}\pi D_{cu}h_{cu})^{1/2} \quad \text{and} \quad C_2 = (k_{co}\pi D_{co}h_{co})^{1/2}.$$

Thus, the copper and the constantan contribution can be subtracted out using the correlation for heat transferred to cylinders found in McAdams [19].

TRANSFERT THERMIQUE ENTRE PARTICULE ET FLUIDE DANS UN LIT FLUIDISE ET POUR DES PARTICULES UNIQUES

Résumé—On décrit une technique de réponse en fréquence pour évaluer les coefficients de transfert thermique entre un gaz et une particule pour des sphères solides et des lits fluidisés de particules. A des nombres de Reynolds faibles, le nombre de Nusselt de la particule unique approche 2 comme prévu. Néanmoins, la valeur générale pour des lits fluidisés de particules fixes est 20 à 100 fois celle des systèmes de lit fixe où de particule unique, ce qui est en contradiction avec les données connues. Les présents résultats sont plus rationnels mais ils sont dépendants du modèle comme tous les autres résultats de lits fluidisés.

WÄRMEÜBERGANG VOM FLUID AN PARTIKEL IN EINEM WIRBELBETT UND AN EINZELNE PARTIKEL

Zusammenfassung—Es wird eine Frequenzgang-Methode zur Ermittlung des Wärmeübergangskoeffizienten zwischen Gas und Partikeln, Gas und festen Kugeln und zwischen Gas und einem Wirbelbett aus feinkörnigen Partikeln beschrieben. Bei kleinen Reynolds-Zahlen liegt die Nusselt-Zahl für Einzelpartikel erwartungsgemäß nahe bei 2. Im Gegensatz zu den Angaben aus der Literatur ist der Wert für Wirbelbetten aus feinkörnigen Partikeln jedoch 20–100 mal größer als derjenige für Festbetten oder Einzelpartikel. Die hier vorgestellten Ergebnisse sind einleuchtender, sie sind aber–wie bei allen anderen Ergebnissen von Wirbelbetten—modellabhängig.

ТЕПЛОПЕРЕНОС ОТ ГАЗА К ЧАСТИЦЕ В ПСЕВДООЖИЖЕННОМ СЛОЕ И В СЛУЧАЕ ЕДИНИЧНЫХ ЧАСТИЦ

Аннотация—Описывается применение метода частотных характеристик для определения коэффициентов теплопереноса от газа к частице в случае твердых сфер и псевдоожиженных слоев, состоящих из мелких частиц. Как и ожидалось, при низких числах Рейнольдса число Нуссельта для единичной частицы приближается к 2. Однако его значение для псевдоожиженных слоев мелких частиц в 20–100 раз больше, чем в системах упаковнных слоев или единичных частиц, что резко отличается от имеющихся в литературе данных. Полученные результаты являются более надежными, хотя и зависят от модели, как и все результаты для псевдоожиженных слоев.